

## The Pendulum and Error Analysis

*The purpose of this lab is to find empirically the law that connects the period  $T$  of a pendulum to its length  $L$ . You apply the rules of “Error and Uncertainty” to calculate errors of measured quantities. We learn the use of computerized lab equipment, helpful for later experiments too.*

### Equipment

- 1 Pendulum: 1 steel ball, 1 holder, 1 string
- 1 Protractor (to measure angles)
- 1 Computer: to be used as a simple timer
- 1 Ruler: (to measure length)
- 1 MANUAL with Report Sheets

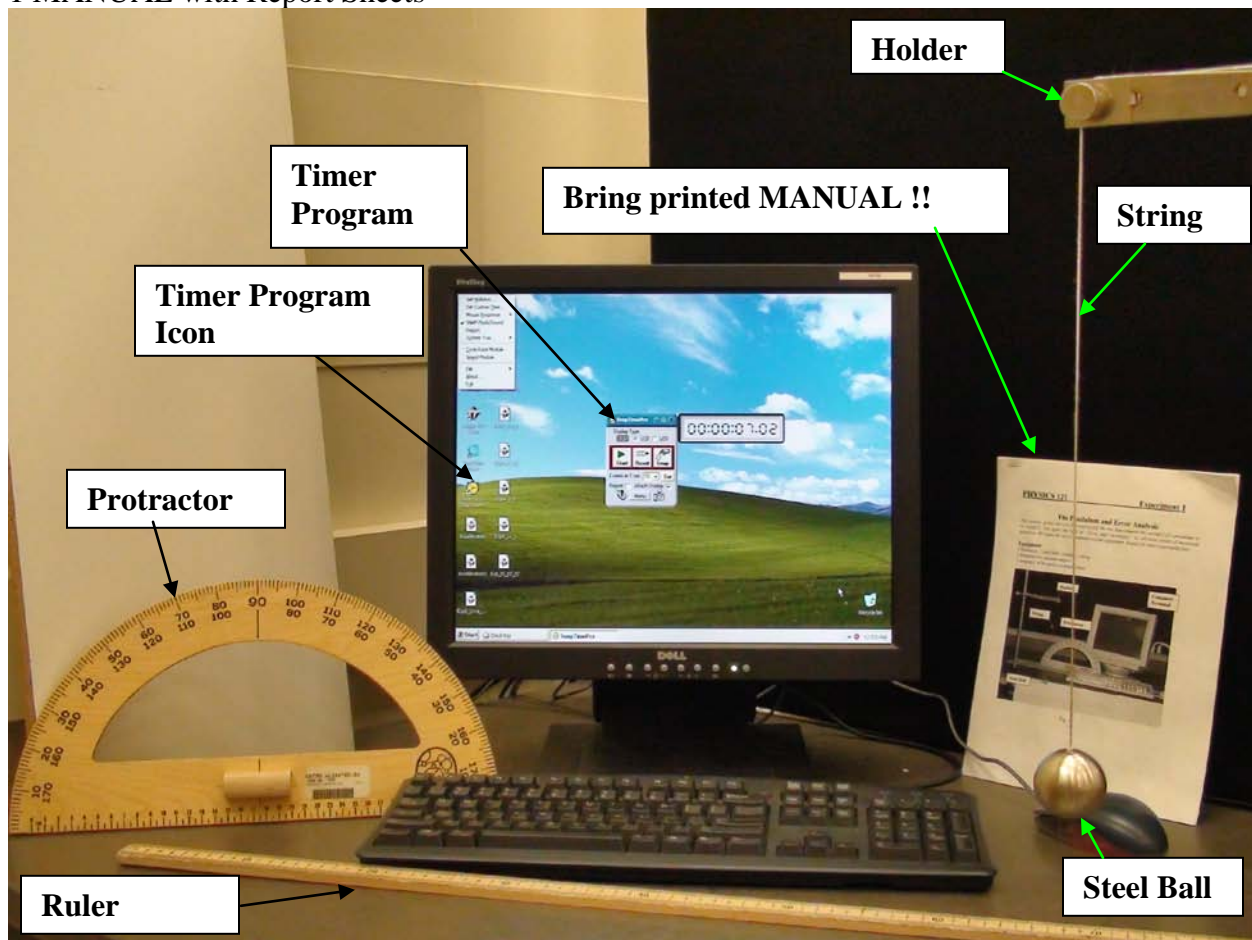


Fig. 1

## Introduction

A simple pendulum consists of a weight  $w$  suspended from a fixed point by a string of length  $L$ . The weight swings about a fixed point. At a given time,  $\theta$  is the angle which this string makes relative to the vertical (direction of the force of gravity).

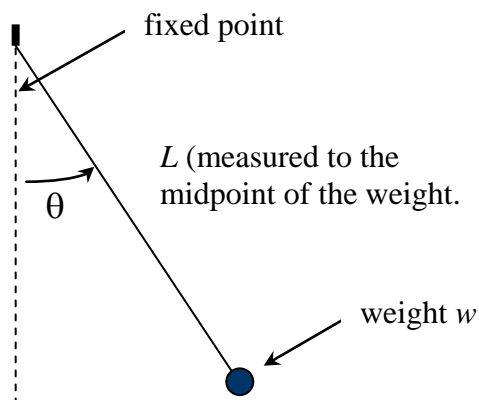


Fig. 2

The **period** of this motion is defined as the time  $T$  necessary for the weight to swing back and forth once.

### **Part I Measurement of the Random Error in the Length $L$ :**

In this part you apply the formulae (5) and (5') in “**Error and Uncertainty**” (“EU”). The random error of  $L$  can be estimated by repeated measurements of  $L$ .

#### **Procedure**

Pick a length of  $\sim 50$  cm measure  $L$  5 times and enter your values into Table 1 in your **REPORT SHEET**.

Use expressions (5) and (5') in “EU” to calculate the average of  $L$  and its error ( $\Delta L_{\text{avg}}$ ) from your 5 measurements.

### **Part II Demonstrate that the Period $T$ varies with Large Angle $\theta$ :**

Choose three angles at  $\sim 15^\circ$ ,  $\sim 30^\circ$  and  $\sim 80^\circ$  and measure the time for 10 periods for each angle and enter your data into Table 2 in your **REPORT SHEET**.

Use the **computer** as a simple **timer**: Double click the Timer Program Icon. The Timer Program comes up. It has a “Start” “Stop” toggle button.

**Estimate** the error of  $T$  using  $2/10$  of a second for your response time. Note that you have this error for the start and end time of the oscillation and that you must propagate these errors into the error of the difference between end time and start time according to expression (6) in “EU”. Note that you have to divide your measured time by 10, in order to get the period  $T$ , and hence must consider expression (1) of “EU”.

**Q1:** Does the period vary with the angle  $\theta$ ? Note that experimentally two values  $T_1$  and  $T_2$  are equal if their ranges  $[T_1 - \Delta T_1, T_1 + \Delta T_1]$  and  $[T_2 - \Delta T_2, T_2 + \Delta T_2]$  overlap each other.

### **Part III Finding the Relation between $L$ and $T$ Empirically:**

You will learn later in Chapter 9 on oscillations that an approximate relation between the period  $T$  and length  $L$  of the pendulum is given by

$$T = 2\pi\sqrt{l/g} \quad (1)$$

where  $g$  is the constant acceleration of gravity,  $g = 9.81 \text{ m/s}^2$ . In the derivation of this equation in Chapter 9 the assumption is made, that the angle  $\theta$  is small. Thus we use the angle of  $\sim 15^\circ$  in this quantitative Part III.

### **Procedure**

Use angles of approximately  $15^\circ$ . Measure for various lengths  $L$ , from  $\sim 10 \text{ cm}$  to  $\sim 1 \text{ m}$  in intervals of  $\sim 10 \text{ cm}$ , the times for 10 periods  $T$  each. Enter your values into Table 3 in your **REPORT SHEET**. Use for the error of  $(10T)$  your estimate in Part II. For the error of your **single** measurement of  $L$ , use for all lengths  $L$  your error  $\Delta L_{\text{avg}}$  in Part I, the error of the average,  $\bar{L}$ , multiplied by  $\sqrt{(N-1)} = \sqrt{(5-1)} = 2$ , and enter them into Table 3 (The factor  $\sqrt{(N-1)}$  turns the error of the average into an error for a single measurement, i.e. turns the expression on the right hand side of equation (5'') into the expression on the right hand side of equation (5').)

**Q2:** Why did you not use the  $30^\circ$  or  $80^\circ$  angles for this Part ?

### **Analysis:**

Complete your Table 3 with the values for  $L^2$  and  $T^2$  and propagate the errors of  $L$  and  $T$  into the errors of  $L^2$  and  $T^2$  according to expression (8) in "EU". Note that expression (8) holds for **relative errors**. You, however, have at this point **absolute errors** of  $L$  and  $T$ . Thus you must **convert** those into relative errors according to expression (2) in "EU" before you use expression (8). When you **graph** your measurements you have to include **error bars**. These are **absolute errors** and hence you must convert your results from expression (8) again to absolute errors according to expression (4) in "EU".

After you have completed Table 3 plot the three graphs  $L$  vs  $T^2$ ,  $L^2$  vs  $T$  and  $L$  vs  $T$  including the error bars on both axes (NOTE you may neglect error bars which are smaller than your plotted points)

**Q3:.** Are the errors on  $L$  and  $T$  you got from Parts I, II absolute or relative errors?

**Q4:** For which type of error is expression (8) in "EU" valid, for absolute or relative errors?

**Q5:** Which types of errors do you plot, absolute or relative errors?

**Q6:** After inspecting your three graphs, which one is best represented by a straight line, a “linear” relationship, that is

$$L = kT^2 \text{ or } L^2 = KT \text{ or } L = KT \quad (2)$$

where  $K$  is the slope  $m$  in equation (3) below?

**Q7:** Which of the relations do you expect to represent formula (1) given above. Show explicitly by converting equation (1) into the format

$$y = mx + b \quad (3)$$

where  $m$  is the slope and  $b$  the intercept of a straight line in an  $y$  vs  $x$  plot. (Hint: you must take the square of equation (1))

Get the slope  $m$  from your graph which best represents a straight line according to expression (10) in “EU” and determine the error of the slope according to expression (11) in “EU”.

**Q8:** How is the acceleration of gravity in equation (1) related to your measured slope  $K$ ?

**Q9:** Using your relation from Q8 what is your measured acceleration of gravity  $g$  and its error according to expression (1) in “EU”? Compare it to the expected value of  $9.81 \text{ m/s}^2$ , that is: call your measured value  $g_m$  and check whether the expected value of  $9.81$  falls into the range  $[g_m - \Delta g_m, g_m + \Delta g_m]$ .

**PHY 121**  
**EXPERIMENT 1**

**REPORT SHEET ( to be signed by instructor)**  
**Pendulum**

Name: \_\_\_\_\_ Section: \_\_\_\_\_

SB#: \_\_\_\_\_ Date: \_\_\_\_\_

Labinstructor: \_\_\_\_\_

**Part I Measurement of the Random Error in the Length  $L$ :**

Table 1:

Measurement #	L [ ]
1	
2	
3	
4	
5	
$L_{avg} \pm \Delta L_{avg}$	

Calculate the average of  $L$  and its error here (show your calculation explicitly)

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**Part II Demonstrate that the Period  $T$  varies with Large Angle  $\theta$ :**

Table 2:

Angle	10T [ ]	T [ ]	$\Delta T$ [ ]
$\sim 15^\circ$			
$\sim 30^\circ$			
$\sim 80^\circ$			

Calculate the error of  $T$  for the  $15^\circ$  angle here (show your error propagation explicitly); assume that the error is the same for all three angles and enter them into the Table 2.

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[      ]

**Q1:** \_\_\_\_\_

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**Part III Finding the Relation between  $L$  and  $T$  Empirically:**

Table 3:

$L$ [      ]	$\Delta L$ [      ]	$10T$ [      ]	$T$ [      ]	$\Delta T$ [      ]	$T^2$ [      ]	$\Delta T^2$ [      ]	$L^2$ [      ]	$\Delta L^2$ [      ]

Propagate for the first measurement your error in  $L$  into the error of  $L^2$  and do the same for  $T$  and  $T^2$ . You must show this calculation explicitly.

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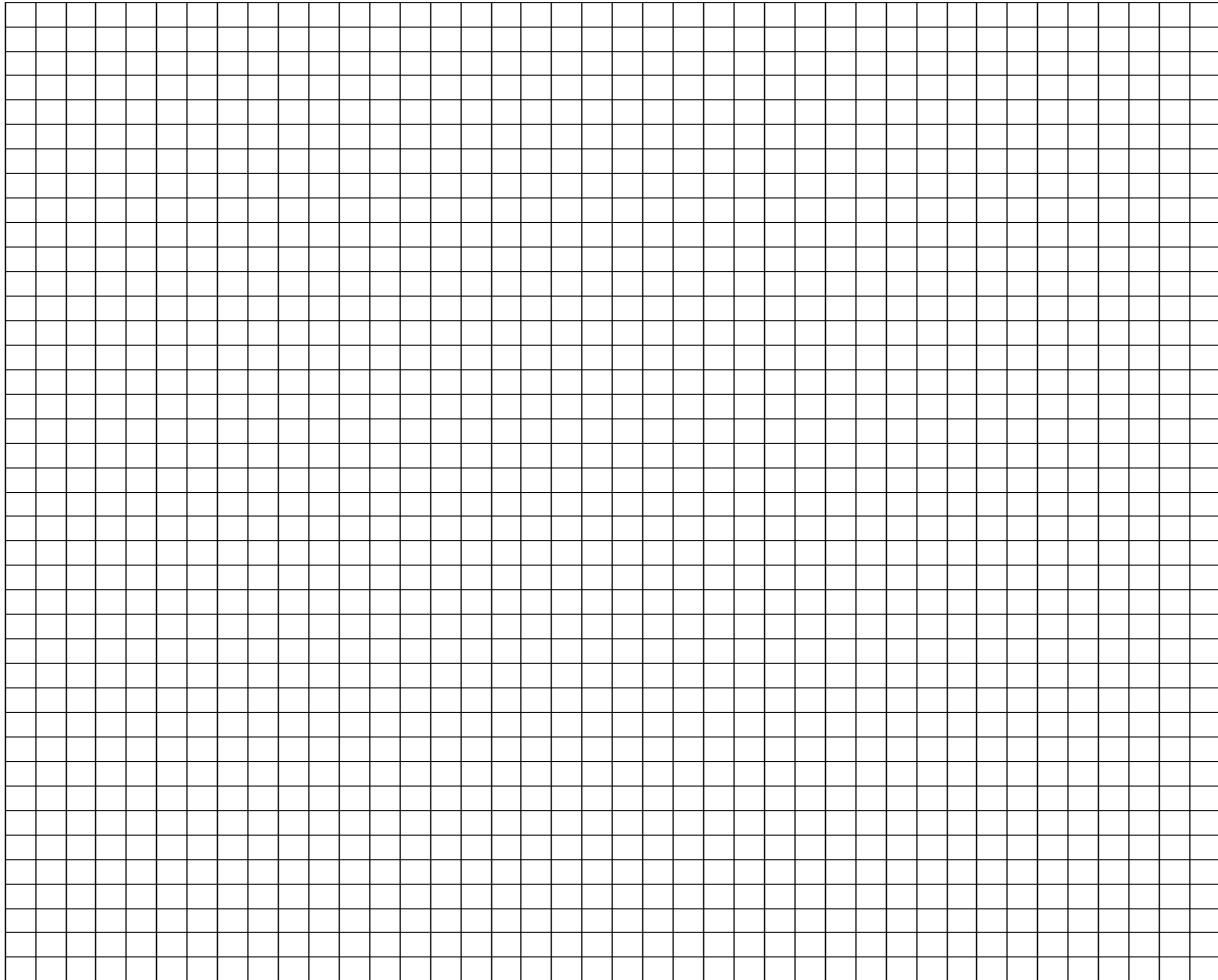
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Q2:

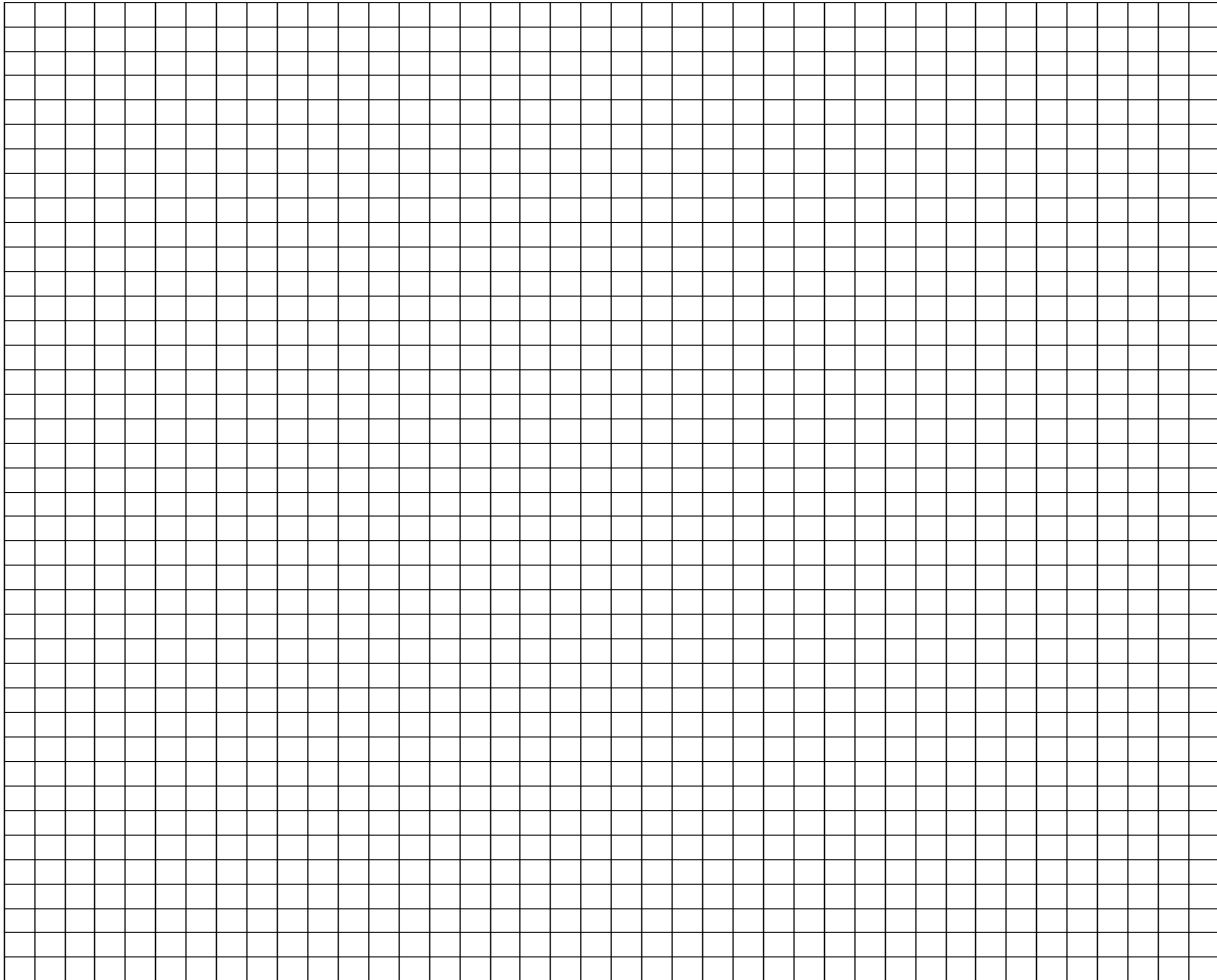
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L vs.  $T^2$  (label all axes and units, if this the correct graph, then mark the ranges used for calculation of slope):

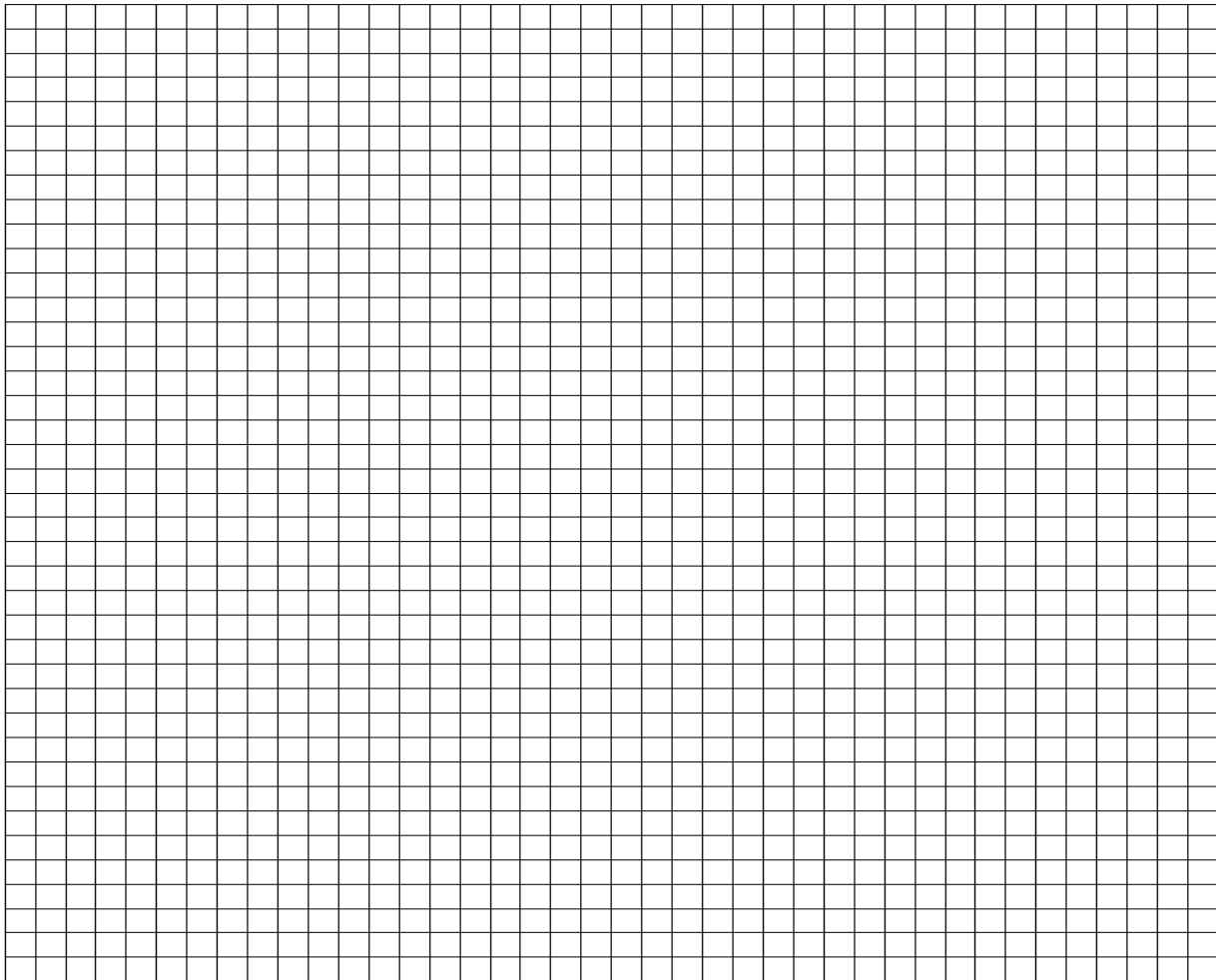


$L^2$  vs. T (label all axes and units, if this is the correct graph, then mark the ranges used for calculation of slope):





L vs. T (label all axes and units, if this is the correct graph, then mark the ranges used for calculation of slope):



Q3: \_\_\_\_\_

Q4: \_\_\_\_\_

Q5: \_\_\_\_\_

\_\_\_\_\_

Q6: \_\_\_\_\_

Q7: \_\_\_\_\_

Determine the **slope K** of your chosen graph. Mark the **ranges  $\Delta y$**  and  **$\Delta x$**  in your graph which you use for the calculation of the slope and show your calculation explicitly.

\_\_\_\_\_ [      ]

Calculate the error in the slope. (Give your explicit calculation) \_\_\_\_\_

\_\_\_\_\_ [      ]

Q8: \_\_\_\_\_

\_\_\_\_\_

Q9 (show your calculations explicitly): \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_ [      ]

The expected value of g \_\_\_\_\_